1997-98 School Year

Mathematics 33 Information Bulletin

Diploma Examinations Program

Students First!
Student Evaluation

Alberta
EDUCATION

This document was written primarily for:

Students	✓
Teachers	✓ Mathematics 33 teachers
Administrators	✓
Parents	
General Audience	
Others	

Distribution: Superintendents of Schools • School Principals and Teachers • The Alberta Teachers' Association • Alberta Education • General Public upon Request

This document can be found on our Web pages. Our Internet address is http://ednet.edc.gov.ab.ca.

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Purpose of the Bulletin

The purpose of this bulletin is to provide students and teachers of Mathematics 33 with information about the diploma examinations scheduled for the 1997–98 school year.

This bulletin includes descriptions of the Mathematics 33 diploma examinations, that will be administered in November 1997, January, June, and August of 1998, descriptions of the acceptable standard and the standard of excellence, and subject specific information.

Teachers are encouraged to share the contents of this bulletin with students.

As well, the publication *Students First: A Guide for Students Preparing to Write the Mathematics 33 Diploma Examination* contains significant information for students.

The document entitled *General Information Bulletin, Diploma Examinations Program* (1997–98) provides administrative information about the diploma examinations for the 1997–98 school year.

This bulletin, the *General Information Bulletin*, and the *Students First* guide are distributed to all senior high school principals and are also available on the Internet at Alberta Education's Web site (http://ednet.edc.gov.ab.ca).

If you have questions or comments about the contents of this bulletin, please contact one of the following persons:

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Important Dates in the 1997–98 School Year

Administration of the Mathematics 33 Diploma Examination

The diploma examination is designed for a writing time of 2.5 h. Students must be allowed an additional 0.5 h to complete the examination if required; therefore, examination supervisors must be available for this additional time.

1997–98 Administrations	*Time*
Thursday, November 6	9:00–11:30 A.M.
Tuesday, January 27	9:00–11:30 A.M.
Wednesday, June 24	9:00–11:30 A.M.
Wednesday, August 12	9:00–11:30 A.M.

Please consult the *General Information Bulletin*, *Diploma Examinations Program*, for a complete schedule for all subject areas.

Scoring of the Examination

Scoring of the examination will take place shortly after the administration date.

In early September, the Student Evaluation Branch sends superintendents a letter requesting the names of recommended markers. In this letter, dates for scoring the examination will be announced.

Student Evaluation Branch staff will contact recommended markers before each marking session to confirm participation, and the dates and times for marking.

Notes of Interest

Additional Diploma Examination Administrations

Alberta Education is increasing the number of administrations of diploma examinations to accommodate students' examination needs more effectively. For the 1997–98 school year, in addition to the examinations already offered in January, June, and August, the following diploma examinations will be offered in November and April.

Tuesday, November 4, 1997	9:00–11:30 A.M.	English 30 Part A English 33 Part A
Wednesday, November 5, 1997	9:00–11:00 A.M.	English 30 Part B English 33 Part B
	1:00–3:30 P.M.	Chemistry 30
Thursday, November 6, 1997	9:00–11:30 A.M.	Mathematics 30 Mathematics 33
Monday, April 20, 1998	9:00–11:30 A.M.	Social Studies 30 Social Studies 33
	1:00–3:30 P.M.	Physics 30
Tuesday, April 21, 1998	9:00–11:30 A.M.	Biology 30

The November and April examinations will be available only to students who have school marks in the courses and to individuals having mature student status.

All students must preregister to write diploma examinations in November and April; walk-ins will not be allowed. Students should contact their school principal for application forms and for the location of the nearest writing centre. The application deadlines for the November and April writings are October 10, 1997, and March 26, 1998, respectively.

Data Resources for Students

A separate data booklet has been provided for the Mathematics 33 Examination. The booklet contains a formula sheet, 90% Box Plots, loan tables, mortgage tables, annuity tables, and present value of an annuity tables. The Mathematics 33 Course of Studies recommends using tables and spreadsheets rather than calculating annuities using a geometric series. Therefore, examination items will be related to recommended tables and spreadsheets. The formulas contained in the data booklet have been recommended by Mathematics 33 teachers across the province, and ongoing feedback is encouraged.

Student Use of Scientific Calculators

Examinations are constructed to ensure that the use of particular scientific calculators neither advantages nor disadvantages individual students. Students are expected to provide their own scientific calculators.

Refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations. Students should be made aware of this policy as early as possible in the school term to ensure they are able to use the scientific calculator of their choice when writing examinations. It is important to clarify with students exactly what they are allowed to have stored in their calculator memories.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations (see the *General Information Bulletin*).

Student Requests for Rescores

Students may request a rescoring of their diploma examinations. Before applying for a rescoring, students should check their *Diploma Examination Results Statement* to find out the distribution of marks. The mark on the machine-scored questions is not likely to change as a result of a rescore, but the written-response mark could change slightly. Students should remember that the rescored mark will be the final mark whether it *increases* or *decreases*.

Students who decide to have an examination rescored must ensure that their application is received before the deadline specified on the results statement. The fee for rescoring each examination is $26.75, which includes GST. If a diploma examination mark is increased by 5% or more as a result of rescoring, the fee is refunded.

Student Requests for Rewriting

Students may rewrite a diploma examination in any regularly scheduled administration. The fee for rewriting each examination is $26.75, which includes GST. Please note that this rewrite fee applies to students repeating the course as well as to those who choose to write the examination. Students must apply to the Student Evaluation Branch by the following dates to be eligible to rewrite the diploma examinations.

Examination Administration	**Rewrite Request Deadline**
November 1997	October 10, 1997
January 1998	November 14, 1997
June 1998	April 29, 1998

(For more details, see the *General Information Bulletin*.)

Children in Need of Protective Services

If a student who is under 18 years of age indicates in his or her writing that he or she is a victim of physical or sexual abuse, the markers are required *by law* to refer the paper to Social Services. Social Services is required to investigate situations that suggest that children might require protection.

Information for Markers

The written-response questions of the Mathematics 33 Diploma Examinations are marked by Mathematics 33 teachers selected from those who have been recommended as markers to the Student Evaluation Branch by their superintendents.

Qualifications

To qualify for recommendation, a teacher must have taught the complete Mathematics 33 course for two or more years (or four semesters) prior to the current school year, be teaching the course in the current school year, and have an Alberta or Northwest Territories Permanent Professional Teaching Certificate.

Dates for Recommendation

Teachers who wish to be recommended as markers for the diploma examinations should contact their superintendents by the following dates.

Diploma Date	Contact Superintendent by
January 1998	October 01, 1997
June & August 1998	March 01, 1998

Markers for the November and April examinations will be selected from lists of prospective markers submitted by their respective school superintendents.

Criteria for Selection

More teachers are recommended as markers by superintendents than are required by the Student Evaluation Branch for any one marking session. The following criteria are considered when markers are selected for a particular marking session:

- experience as a marker (experienced markers and first-time markers are included)
- regional representation (by zone, jurisdiction, and school)
- student population (proportional representation)

We particularly need teachers who can mark examinations written in French.

Examination Development

As the need arises for teachers to participate in field testing and item writing, letters are sent to superintendents requesting their nominations. Item writing takes place throughout the year and is a vital component of the diploma exam program. Field testing provides students and teachers with the opportunity to become familiar with the nature of questions they will encounter on future diploma exams. Only teachers who have been nominated by their superintendents are eligible to participate in these activities. Therefore, it is essential that teachers who are interested in these activities should contact their superintendents.

Inservices and Presentations

On a limited basis and subject to budget constraints, Student Evaluation Branch staff may be available to provide inservices or presentations related to diploma examinations or the interpretation of diploma examination results.

Other Important Documents

Guide for Students

A publication written for students called *Students First: A Guide for Students Preparing to Write the Mathematics 33 Diploma Examination* is available in each senior high school and on Alberta Education's Web site (http://ednet.edc.gov.ab.ca). This publication contains suggestions for exam preparation and exam writing suggestions. The Student Evaluation Branch highly recommends that students and teachers examine this document early in the term to prepare for the Mathematics 33 Examination. Parents will also find this to be a useful publication.

Examiners' Reports

Following the administration of each January and June diploma examination, the Student Evaluation Branch sends copies of the *Mathematics 33 Examiners' Report* to all senior high schools in Alberta, and also posts it on Alberta Education's Web site (http://ednet.edc.gov.ab.ca).

The *Examiners' Report* contains

- detailed provincial results
- examiners' comments about student performance on the exam
- a detailed blueprint of the examination by reporting category
- the key and difficulty level for all the questions on the examination
- sample questions accompanied by commentary

The *Examiners' Report* provides important information for teachers who wish to interpret their students' results in relationship to provincial results as an ongoing form of program evaluation.

School and Jurisdiction Reports

School and Jurisdiction Statistical Reports are made available to schools and school jurisdictions after each January and June administration of the diploma examinations. These reports are placed on the Dial-In Facility of the Educational Information Exchange and provide detailed information on how well the students in their school and school district, respectively, did on each of the diploma examinations. Teachers may use these data to reflect on the areas of the program where their students did well and those areas where student performance was poor.

Annual Reports

Each year, the Student Evaluation Branch produces the *Annual Report, Diploma Examinations Program*. This report contains information about the results achieved by all students who wrote diploma examinations in the preceding school year. It also contains special studies on topics of interest.

Each of these special studies will be of interest to teachers who are involved in interpreting the diploma examinations results of their students.

Diploma Examinations Program

Year	Special Studies Topic
1989–90	Comparing Achievement in Various Diploma Examination Courses
1990–91	Different Paths to Success in Diploma Examination Courses
1991–92	Participation Rates in Diploma Examinations Courses
1992–93	Conventions of Language
1993–94	Participation Rates Over Time
1994–95	Grade 12 Enrollment Decline—A Preliminary Assessment
1995–96	Differential Item Performance Between Males and Females on the June 1996 Administration of the Social Studies 30 Diploma Examination

The 1995–96 Annual Report is available on the Internet at Alberta Education's Web site (http://ednet.edc.gov.ab.ca).

General Information Bulletin

Each September, the Student Evaluation Branch sends all senior high school principals and superintendents copies of the *General Information Bulletin, Diploma Examinations Program*, and also posts it on Alberta Education's Web site (http://ednet.edc.gov.ab.ca). This bulletin provides the information, dates, policies, and rules that apply to the administration of all diploma examinations for the school year.

Teachers and students who have questions about such issues as partial writings, word-processor policy details, special provisions for learning and/or physically disabled students writing diploma exams, dates for exam administration, and definitions of mature students, should consult this bulletin.

Previous Examinations

Previous Mathematics 33 January and June diploma examinations are posted on the Alberta Education's Web site (http://ednet.edc.gov.ab.ca) and are also available for purchase (@$4.00 + G.S.T.) from the

Learning Resources Distributing Centre
12360-142 Street, Edmonton, AB T5L 4X9
Phone: 403-427-5775 (toll-free 310-0000) Fax: 403-422-9750

Teachers and students who are consulting previous exams should use exams administered under the present *Program of Studies*.

Mathematics 33 Information

Standards

Standards are statements that communicate how well a student needs to perform in order to be judged as having met the required learnings specified for Mathematics 33. Accordingly, the learnings for Mathematics 33 are referred to in the *Mathematics 13–23–33 Senior High Program of Studies* as the specific knowledge, skills, and attitudes that students should have. These learnings are amplified as standards in Appendix B, Mathematics 33 Curriculum Standards. The standards are applicable to students who achieve the *acceptable* standard of achievement (who receive a final course mark of between and including 50% and 79%) and students who achieve the *excellent* standard of achievement (who receive a final course mark of 80% or better). Specific statements of standards in Appendix B are written primarily to inform teachers about the extent to which students must know Mathematics 33 content and must demonstrate the skills to pass the examination. Examples of questions are provided to give a profile of the *acceptable* and *excellent* standards of achievement.

Examination Specifications

In keeping with the expectations listed in the *Senior High Program of Studies for Mathematics 33*, the 1997–98 examinations will reflect the need to understand ideas that make up concepts and the relationships between concepts. Most importantly, the examinations will reflect the need to apply understanding of concepts and procedures to solving problems.

Each Mathematics 33 diploma examination is designed to reflect the general and specific learner expectations outlined in the *Senior High Program of Studies for Mathematics 33*. Some questions will assess specific learner expectations, whereas other questions will be based on the integration of several learner expectations. In relation to curriculum content and the general outcomes specified in the curriculum standards document, questions may be presented that assess students' overall understanding in content areas.

As well, a question or set of questions may assess understanding from a number of core content areas. Questions are organized into sets for a variety of reasons. Sets of questions are used to provide context and to connect the areas of understanding that students must have. An individual set of questions is referred to as a scenario, and any examination contains as many as five scenarios.

Questions organized into scenarios also assess the students' ability to realize general learner expectations. In this, students need to use their mathematical understandings to apply, analyze, investigate, interpret, and solve problems related to real-world phenomena such as future endeavors and employment situations.

As the scope of Mathematics 33 includes pure and applied mathematics connected to previous understandings, a scenario named "Connections" is used. This scenario challenges students to make and apply mathematical connections. This scenario provides an opportunity for students to show that they understand the connections and interplay among various mathematical concepts and can also link mathematical understandings to real-world situations and to other disciplines.

Based on the premise of what students need to learn and be able to do, the scenario format of the examination reflects students' ability to confidently transfer known mathematical understandings to new or existing practical situations.

Questions are organized on the examination according to the following list of scenario topics:

1. Alberta Industries
2. Connections
3. Consumerism
4. Science and Technology
5. Sports and Recreation

These topics were chosen for their relevance to the current program of studies and to the students' post-secondary needs and opportunities.

Teachers have been involved in writing the scenario questions in applied and pure mathematical contexts. Businesses and post-secondary institutions have been involved in making curriculum connections and providing data for questions.

Format

The 1997–98 examinations will use the same examination format as the 1997 administrations. That is, the examinations will have both machine-scored and teacher-scored (written-response) questions. The machine-scored questions will be of two types: multiple choice and numerical response. Students will record their answers to machine-scored questions on a form like the one shown in Appendix E. Special instructions for filling in answers to multiple-choice, numerical-response, and written-response questions can be found in Appendix F.

The multiple-choice, numerical-response, and written-response questions are embedded in the scenarios. Scenarios are introduced in a bordered area of the examination with a title, caption, and/or picture. The introduction provides a meaningful context to sets of questions and/or refers students to the major understandings under which questions are organized. Within the scenarios, students may find other bordered areas that provide additional information required to solve the questions that follow. A message or caption reminding students to use the information appears above the bordered area.

Some bordered areas may be presented in italics. Italicized areas are intended to provide a link between questions and/or major understandings.

Each examination will contain four written-response questions. Three will be equal in value (5 marks), and the fourth question is worth slightly more (6 marks). Student answers to the written-response questions should be carefully written in the space provided and will be scored by teachers using holistic and anaholistic scoring criteria. The scoring criteria describes the characteristics of an answer, which correspond to a number on a scale. The scale is appropriately proportioned to the task or question being scored and can vary between zero and five marks. Each student's answer will be scored by one teacher. A sample of a question and holistic scoring criteria are provided in Appendix G. The generalized five-mark criteria is one that is adapted and that students should become familiar with.

The format of written-response questions includes the use of directing words for both writing and marking purposes. For an explanation of the directing words used on examinations, see Appendix D.

Examination Design

The Mathematics 33 Diploma Examination is designed to reflect content as outlined in the *Senior High Program of Studies for Mathematics 33* (1991). The examination is limited to those expectations that can be measured by a paper-and-pencil test.

Question Types

The design of the 1997–98 Mathematics 33 diploma examinations are as follows.

Question Format	*Number of Questions*	*Marks*	*Percent Emphasis*
Written Response	4	21	30
Numerical Response	12	12	17
Multiple Choice	37	37	53

Core Content Emphasis

The core content of the course is used to build the examination. The emphasis below is used as much as possible to build the machine-scored part of the examination. The written-response items are used to cross-over and emphasize as many of the content areas as possible.

Core Content	*Percent Emphasis*
Powers and Radicals	10
Annuities, Mortgages, and Loans	10
Statistics	13
Trigonometry	18
Polynomials & Rational Expressions	13
Relations and Functions	18
Quadratic Functions & Equations	18

Mathematical Understandings

The three mathematical understandings of procedures, concepts, and problem solving are also used to build the examination. The suggested emphasis of these understandings is listed below, and again this emphasis is used as much as possible to build the examination.

	Percent Emphasis
Written Response	
Procedures, Concepts, and Problem Solving	30 (combined)
Multiple Choice and Numerical Response	
Procedures	30
Concepts	25
Problem Solving	15

For further information regarding the mathematical understandings above, see Appendix C.

Abstract and Concrete Understandings

In keeping with the pure and applied nature of the Mathematics 33 course, students will be required to demonstrate both abstract and concrete understandings of core content areas. An example of a concrete understanding is being able to apply knowledge gained in the Annuities, Mortgages, and Loans Unit to one's personal financial situation or to operating a small business. Abstract understandings are seen, for example, as being used to connect knowledge of quadratic functions to their graphical representations. Concrete understanding in quadratics would relate to applying an equation or graph to a real-world situation.

Directions for the 1997–98 Examinations

Expectations

Over the recent administrations, Mathematics 33 students have met the expectation of applying their mathematical understandings to various contexts. Some of the problems being solved have involved a multi-step approach, and this will continue to be emphasized. Applying mathematical understandings in context and problem solving are two of the general learning expectations of the course. An attempt will be made to continue to ensure that general learner expectations (as described in the course of studies) and general outcomes (as described in the standards) are emphasized on the 1997–98 examinations.

Communication

During the past year, diploma exam markers have insisted that mathematical statements or procedures be clearly communicated. Teachers' committees have endorsed the general learner expectation that students be able to communicate mathematical ideas clearly and effectively. The open-ended questions will continue to require students to communicate answers effectively in writing. Therefore, students will be required to explain a solution, articulate their reasoning, describe a mathematical situation, write directions to a problem, pose questions, and/or explain new understandings. Included in this, students are expected to provide a concluding statement complete with a correct answer and relevant units.

Problem Solving

The written-response questions focus on communication and students' understanding of the problem-solving process. Students will be rewarded for selecting and carrying through a problem-solving strategy. Students are required to describe their method of problem solving and to communicate their descriptions of mathematical concepts and procedures. Effectively communicating solutions will enhance student grades on the written-response component of the examination. For purposes of the 1997–98 diploma examinations, teachers have asked us to emphasize that the expectation for full credit will be that the solution contains supporting details with no communication or syntax errors.

Appendices

Appendix A

Calculator Policy

POLICY: USE OF CALCULATORS ON ALBERTA EDUCATION DIPLOMA EXAMINATIONS

Rationale

The knowledge, skills, and attitudes relevant to technology and its uses are being incorporated into courses and programs of study wherever appropriate. Students are expected to learn the advantages and limitations of technology developments and their impact upon society. The ability to use technology helps students understand and appreciate the process of technological change, gives added depth to programs, and provides the basis for the development of skills and understanding. These expectations are reflected in the diploma examination. Since the data provided for writing diploma examinations in mathematics and the sciences do not include information such as logarithms and trigonometric functions, students will need to use scientific calculators for these exams.

Definition

This policy considers a scientific calculator to be a hand-held device designed primarily for mathematical computations. Included in this definition are those scientific calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness to all students, Alberta Education expects students to use scientific calculators, as defined above, when they are writing diploma examinations in mathematics and the sciences. Examinations are constructed to ensure that the use of particular models of calculators neither advantages nor disadvantages individual students.

Procedures

1. Teachers must, at the beginning of a course, advise students of the types of calculators that they may use when writing diploma examinations in mathematics and the sciences. Teachers must also advise students of the types of information that can be stored in calculators that are brought into diploma examinations. Calculators that have built-in notes (definitions or explanations in alpha notation) that cannot be cleared are not permitted.

2. Students must clear calculators that are brought into diploma examinations of all information that they have stored except for
    a. programs used for computing values of the formulas on the diploma examination data tear-out pages or in the data booklets
    b. programs used for graphing quadratic relations as found in the Mathematics 30/33 Interim Teacher Resource Manual

3. Students must not bring external devices to support calculators into the exam. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outline operational procedures for scientific calculators.

4. In preparation for calculator failure, students may bring extra calculators and batteries into the exam room.

5. During exams, supervising teachers must ensure that
    a. all calculators operate in silent mode
    b. students do not share calculators or information contained within them
    c. calculator cases are stored on the floor throughout the exam
    d. all examination rules are followed

6. If you have any questions or comments about the implementation of this policy, please contact the Math/Science Unit, Student Evaluation Branch, at 403-427-0010 or FAX 403-422-4200.

Appendix B

Mathematics 33 Standards and Sample Questions

Contents

Curriculum Standards

Curriculum standards are criteria that can be used to judge when a student has met the expectations of the Program of Studies for a course.

Curriculum standards are statements of what students are expected to know and be able to do. These statements are made in the *Senior High Program of Studies for Mathematics 33* and have been enlarged upon in this document in order to set the stage for the development of the Diploma Examination. The statements made here represent the *Program of Studies* and have been organized to include general outcomes for each unit and specific outcomes for each unit. The specific outcomes have been further divided into expectations that will be held for students who demonstrate **acceptable** and **excellent** achievement.

Acceptable Achievement

Students who demonstrate acceptable achievement but not excellent achievement in Mathematics 33 will receive a final course mark between and including 50% and 79%. These students have demonstrated mathematical skills and knowledge in seven content strands of the Mathematics 33 curriculum and exhibit an ability to apply a range of problem-solving skills to these content strands. Typically, they have gained new skills and knowledge in mathematics, but students at the lower end of the acceptable achievement level can anticipate difficulties if they choose to enroll in post-secondary mathematics courses.

Excellent Achievement

Students who demonstrate excellent achievement will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in post-secondary mathematics programs; they should be encouraged to pursue careers in which they will use their talents in mathematics.

Statements

The specific statements of standards that follow were written primarily to inform Mathematics 33 teachers about the extent to which students must know the Mathematics 33 content and must demonstrate the required skills to pass the examination.

Examples

A limited number of examples are provided for each section of the course. The questions provided are by no means exhaustive and have been coded to reflect the acceptable and excellent outcomes and are indicative of the type of questions that students will be presented with in the formal examination. The questions for curriculum standards are organized by unit and contextualized into scenarios. Teachers and students are advised to look at previous copies of the diploma examination to get a sense of how questions from various units are mixed together.

Relations and Functions

General Outcome

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Apply the concepts within relationships and functions to describe real-world phenomena that can be expressed in two variables.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can:**
• interpret, orally or in writing, a graph that models a real-world situation	
• create a graph that models a real-world situation	• illustrate the relationship between two variables in an everyday situation using a defining rule
• identify the dependent and independent variables from the graph of a function	• determine whether or not a cause-and-effect relationship exists between two variables, and in the case of such a relationship, decide which of the two variables is the independent variable
• explain why the independent variable is independent in a given context	
• describe, orally or in writing, the domain and range of any function, except the reciprocal function, by examining its graph or a set of ordered pairs	• state the domain and range of any function, including the reciprocal function, using mathematical notation
• interpolate from graphs of continuous functions	
• extrapolate from graphs of linear functions	• extrapolate from graphs of any functions
• verify whether a point is or is not on the graph of a function	
• determine the values of a function given particular domain values	• given particular values for a function, determine the corresponding values of the domain
• recognize, illustrate, and classify linear, quadratic, cubic, absolute value, first-degree reciprocal, radical, and exponential functions	• recognize and classify real-world situations that are depicted as being a linear, quadratic, cubic, absolute value, first-degree reciprocal, radical, or exponential function
• determine the inverse of a function when provided with the graph of the function	• determine the inverse of a function
• explain and show the effect of a single parameter, a, b, or c, on the graph of $y = c \bullet f(x - a) + b$	• explain, orally or in writing, and show the effects of the parameters a, b, and c on the graph of $y = c \bullet f(x - a) + b$
• participate in and contribute toward the problem-solving process for problems that require the use of functions studied in Mathematics 33	• find the complete solution to problems that require the use of functions studied in Mathematics 33

Illustration of the general outcome

Given an everyday situation, students can construct a graphical model of it, analyze it, and then generalize the model to other situations.

The following are examples of questions from the Relations and Functions Unit that illustrate the standard under the scenario of Consumerism.

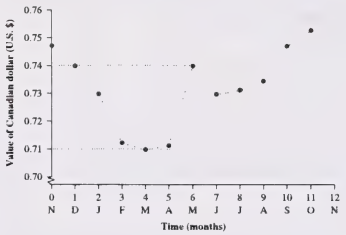
The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONSUMERISM**

Dawn is a financial advisor. She constantly uses mathematics to make decisions related to her customer's investment and banking opportunities. Answer the following questions related to Dawn's experiences.

Use the following information to answer the next two questions.

Dawn often analyzed graphs to determine the best time of the year to exchange currency. The graph below indicates the value of the Canadian dollar in U.S. dollars over a period of 12 months.



From the fourth month (March) to the sixth month (May), the value of the Canadian dollar _____*i*_____ by an amount equal to $_____*ii*_____, relative to the U.S. dollar.

1. The **correct** word for ***i*** is

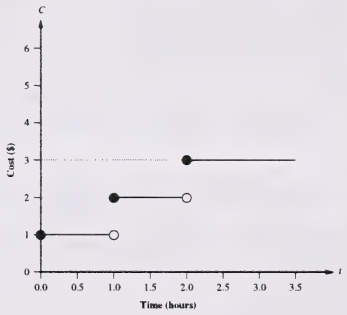
   *A. increased
   B. decreased
   C. amortized
   D. matured

Numerical Response

1. The numerical value for ***ii***, to the nearest hundredth of a dollar, is $__0.03__. (Record your answer on the answer sheet.)

Use the following information to answer the next question.

The graph below represents the cost, in dollars, that Dawn was charged for various lengths of parking time, in hours, at a parkade.



Written Response

1. Write a story describing the amount of money Dawn would pay in the three different time intervals shown.

Possible Solution:

Dawn enters the parkade and automatically owes the attendant $1.00. If Dawn leaves the parkade before one hour elapses, the amount owing will still be $1.00. After one hour, Dawn's cost jumps to $2.00. Dawn owes $2.00 for any time between one and two hours. After two hours, Dawn's cost jumps to $3.00, and this appears to be the maximum cost that Dawn would pay no matter how long the car is parked.

The student demonstrating excellent achievement can answer the following type of question.

SCENARIO: **TECHNOLOGY**

Yin and Henry used a graphing calculator to assist in analyzing, interpreting, and exploring equations and their graphical representations.

Use the following information to answer the next question.

Henry graphed a function where the graph was v-shaped, its vertex was at $(-3, -5)$, and it passed through $(1, -1)$. Henry then wrote the following statements about the graph:

I. If the equation $y = |x|$ is transformed to give the graph described above, the new equation is of the form $y = a|x + 3| - 5$.

II. The transformed graph is a shift of the graph of $y = |x|$ stretched by a factor of 4.

III. The basic graph has been shifted to the left 3 units and down 5 units.

IV. The graph is an example of an absolute value function.

V. The equation of the graph is $y = -2|x - 3| - 5$.

2. Which of the statements about the graph originally described are true?

A. Statements I, II, and IV
***B.** Statements I, III, and IV
C. Statements I, II, III, and IV
D. Statements I, III, IV, and V

Quadratic Functions and Equations

General Outcome

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Connect variables and equations to graphical representations of quadratic functions that model real-world contexts.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can also:**
• sketch the graphs of quadratic functions $y = a(x - h)^2 + k$, where h and k are integers, a is a rational number, and $a \neq 0$	
• transform $y = ax^2 + bx + c$ to $y = a(x - h)^2 + k$, where h and k are integers and a is a rational number, $a \neq 0$	• transform $y = ax^2 + bx + c$ to $y = a(x - h)^2 + k$, where h and k are rational and $a \neq 0$
• given the graph or the equation of a quadratic function, determine its vertex, axis of symmetry, domain, range, maximum or minimum value of the function, and x- and y-intercepts	• given any quadratic function, determine the vertex, axis of symmetry, domain, range, maximum or minimum value, x- and y-intercepts, and its sketch
• given a real-world situation modelled by a quadratic function, place into the appropriate context the maximum or minimum values and the x- and y-intercepts	• derive a quadratic function that models a real-world situation and place into the appropriate context the maximum or minimum values and the x- and y-intercepts
• solve quadratic equations whose solutions are elements of the real numbers	
• determine whether or not a given quadratic equation describes a particular real-world situation	
• recognize that there is a relationship between the x-intercepts of the graph of a quadratic function, the roots of its corresponding equation, and the zeros of the quadratic function	• explain, orally or in writing, the relationship between the x-intercepts of the graph of a quadratic function, the roots of its corresponding equation, and the zeros of the quadratic function
• solve a radical equation with no extraneous roots consisting of a single radical whose solutions are the roots of a quadratic equation	• solve a radical equation that may have extraneous roots with two or more like radicals and whose solution is the root of a quadratic equation
• solve an equation that includes two rational expressions with monomial denominators, whose solution(s) is/are the root(s) of a quadratic or a linear equation	• solve a rational expression equation whose solution is the root(s) of a quadratic or linear equation
• solve problems that involve equations with radicals or rational expressions, whose solutions are the roots of quadratic equations, when the radical or rational equation is given	• solve problems that involve equations with radicals or rational expressions whose solutions are the roots of quadratic equations
• solve simple problems whose solutions are the roots of quadratic equations	
• use the x-intercepts of a quadratic function, where $c = \pm 1$ in $y = c(x - a)(x - d)$, to determine a corresponding quadratic equation	

Illustration of the general outcome

Given any quadratic function, either directly or implied in a problem, and the equation of the function, students can determine the graph of the function and the roots of the corresponding equation **and** can describe, in writing, the relationship between the roots and the graph.

The following are examples from the Quadratic Functions and Equations unit that illustrate the standard under the scenario of Technology.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **TECHNOLOGY**

Robert and Lim used current technologies to assist in analyzing, interpreting, and exploring equations and their graphical representations.

Use the following information to answer the next three questions.

Given specific information about quadratic functions and equations, Robert and Lim used their abilities to write equations and visualize parabolic graphs.

1. The vertex of the graph of the function $f(x) = (x - 3)^2 - k$ is at

***A.** $(3, -k)$
B. $(3, k)$
C. $(-3, -k)$
D. $(-3, k)$

Numerical Response

1. The graph of the quadratic function $f(x) = 5x^2 - 7x + 21$ has a y-intercept of ___21___. (Record your answer on the answer sheet.)

2. If a quadratic function has a maximum value of h and its graph has an axis of symmetry of $x = 3$, then the function could be

A. $f(x) = -(x + 3)^2 + h$
B. $f(x) = -(x - h)^2 + 3$
***C.** $f(x) = -(x - 3)^2 + h$
D. $f(x) = -(x + h)^2 + 3$

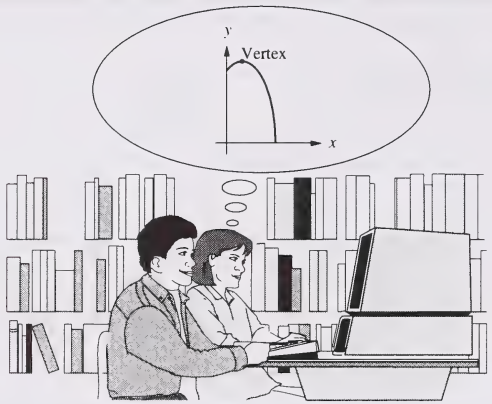
The student demonstrating excellent achievement can answer the following type of question.

3. If two roots of a quadratic equation are $\frac{2}{3}$ and -1, then a possible quadratic equation is

*A. $3x^2 + x - 2 = 0$
B. $3x^2 - x - 2 = 0$
C. $2x^2 - x - 3 = 0$
D. $2x^2 + x - 3 = 0$

Students demonstrating acceptable and excellent achievement can answer part or all of the following question.

Use the following information to answer the next question.



Samantha and Sydney were looking for the important points related to the graph of $y = -2x^2 + 8x + 10$. When determining the vertex, Samantha began completing the square as shown below.

Step 1: $y = -2x^2 + 8x + 10$
Step 2: $y = -2(x^2 - 4x + \_\_\_) + 10$
Step 3: $y = -2(x^2 - 4x + 4) + 10 + 8$
Step 4: $y = -2(\_\_\_\_\_)^2 + \_\_\_\_\_$
Step 5: Vertex is (____ , ____)

Written Response – 6 marks

1. **a.** Finish Samantha's work in steps 4 and 5 by filling in the blanks below.

Step 4: $y = -2(\_\_\_\_\_)^2 + \_\_\_\_\_$
Step 5: Vertex is (____,____)

Possible Solution:

Samantha's work can be completed by factoring $x^2 - 4x + 4$ and determining the value of k in the function $y = a(x - h)^2 + k$:

Step 4: $f(x) = -2(\underline{x - 2})^2 + \underline{18}$
Step 5: Vertex is (**2, 18**)

b. What information from the equation would lead you to conclude the graph opens downward?

Possible Solution:

The graph opens downward because the value of a is negative. Generally, when $a < 0$ in the function $y = -ax^2 + bx + c$, the graph of the related parabola opens downward.

c. Show algebraically where the graph of $y = -2x^2 + 8x + 10$ crosses the x-axis. Start your work by stating what the value of y is at the x-intercepts.

Possible Solution:

The graph crosses the x-axis when the y value is zero. By substituting 0 into the function for y, we obtain $0 = -2x^2 + 8x + 10$, and by factoring, we obtain:

$0 = -2(x - 5)(x + 1)$ **or** $0 = (-2x - 2)(x - 5)$
$\therefore x = 5$ or -1 $\therefore x = -1$ or 5
$\therefore$ x-intercepts are -1 or 5 $\therefore$ x-intercepts are -1 or 5

The x-intercepts are the points $(-1, 0)$ and $(5, 0)$.

or

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to obtain the roots of the equation.

$$x = \frac{-8 \pm \sqrt{64 - 4(-2)(10)}}{2(-2)}$$

$$x = \frac{-8 \pm \sqrt{64 + 80}}{-4}$$

$$x = \frac{-8 \pm 12}{-4} = 5 \text{ or } -1$$

$\therefore$ the x-intercepts are -1 or 5

or

$$0 = -2(x - 2)^2 + 18$$

$$-18 = -2(x - 2)^2$$

$$9 = (x - 2)^2$$

$$\pm 3 = x - 2$$

$x - 2 = 3$ or $x - 2 = -3$
$x = 5$ $x = -1$

$\therefore$ the x-intercepts are -1 or 5

$\therefore$ the graph crosses the x axis at $(-1, 0)$ and $(5, 0)$

Powers and Radicals

General Outcome
Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Determine equivalent forms of radical expressions. Solve and verify radical equations.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can also:**
• perform the operations of addition, subtraction, multiplication, and division on second-order radicals with monomial denominators and numerical radicands, and show that these operations are a specific instance of the operations on polynomials	
• change second-order radicals from mixed to entire form and vice versa	• change third-order radicals from mixed to entire form and vice versa
• solve second-order radical equations	• solve second-order radical equations whose solutions involve the simplification of a quadratic equation
	• solve problems involving radical equations

Illustration of the general outcome
Show that mathematical operations on radicals are similar to the mathematical operations on polynomials.

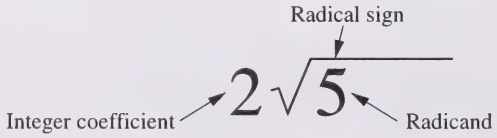
The following are examples from the Powers and Radicals unit that illustrate the standard.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**

Whenever Michelle did peer tutoring, she tried to link new learnings with previous mathematics studied. For example, Michelle linked factoring and procedures learned to simplify fractions to show how rational expressions can be simplified.

In explaining operations involving radicals, Michelle used the illustration below to describe parts of a radical term.



1. When $\sqrt{18}$ is expressed as a mixed radical in simplest form with an integer coefficient, the **radicand** is

   A. $\sqrt{3}$
   ***B.** 2
   C. 3
   D. 9

2. In simplified form, the integer coefficient of $3\sqrt{8} + 2\sqrt{18} - 4\sqrt{98}$ is

   A. 16
   B. 1
   C. −1
   ***D.** −16

To solve a problem related to an accident scene presented in a legal studies textbook, Michelle connected her understanding of radical equations to the following problem.

Numerical Response

1. At the scene of an accident, police measure the approximate speed that a vehicle was travelling by measuring the length of the skid marks left on the pavement. One formula used for this purpose is $v = 14.9\sqrt{L} - 20.4$, where v represents the speed of the vehicle and L represents the length of the skid marks in metres. A vehicle is travelling on a road at 55 km/h and brakes so that it skids to a stop. The length of the skid marks, to the nearest tenth of a metre, that the vehicle will leave on the road is __25.6__ m.

(Record your answer on the answer sheet.)

Solution:

$$55 = 14.9\sqrt{L} - 20.4$$

$$75.4 = 14.9\sqrt{L}$$

$$\frac{75.4}{14.9} = \sqrt{L}$$

$$25.6 = L$$

The length of the skid marks, to the nearest tenth of a metre, is 25.6 metres.

The student demonstrating excellent achievement can answer the following types of questions.

Numerical Response

1. Michelle determined the largest root of the equation $\sqrt{x} + 4 = x - 8$ to be ____16____.

(Record your answer on the answer sheet.)

Solution:

$$\sqrt{x} + 4 = x - 8$$
$$\sqrt{x} = x - 12$$
$$\left(\sqrt{x}\right)^2 = (x - 12)^2$$
$$x = x^2 - 24x + 144$$
$$0 = x^2 - 25x + 144$$
$$0 = (x - 9)(x - 16)$$
$$x = 9 \quad \text{OR} \quad x = 16$$

$x = 9$ is not a solution.

The largest root is 16.

2. When $2\sqrt[3]{5}$ is expressed in the form $\sqrt[3]{a}$, $a > 0$, the radicand is

A. 10
B. 25
C. 32
***D.** 40

Polynomials and Rational Expressions

General Outcome

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Show how the operations on rational numbers can be extended to rational expressions and the applications of rational expressions.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can also:**
• recognize and give examples of rational expressions	
• determine the real non-permissible values of a variable in a rational expression with its denominator in the form of a monomial, binomial, or trinomial of the form $x^2 + bx + c$ or with its denominator in factored form	• determine the real non-permissible values of a variable in any rational expression
• recognize and give examples of rational expressions where there are no real non-permissible values for the variable	• describe the relationship between non-permissible values of a rational expression and the graph of the rational function (and how this relates to the reciprocal function studied in relations and functions)
• simplify rational expressions that contain monomials, binomials, or trinomials in the form $ax^2 + bx + c$ but that do not contain factors of -1	• simplify rational expressions that do contain factors of -1
• multiply or divide two rational expressions that contain monomials, binomials, or trinomials in the form $ax^2 + bx + c$	• multiply or divide two or more rational expressions
• add or subtract two rational expressions with non-factorable binomial denominators	• add or subtract two rational expressions
• solve equations involving rational expressions with non-factorable denominators	• solve equations involving rational expressions with factorable denominators
• solve problems involving rational expressions given the equation	• solve problems involving equations containing rational expressions

Illustration of the general outcome

Show that the operations on rational expressions are a generalization of the operations on rational numbers.

The following are examples from the Polynomials and Rational Expressions Unit that illustrate the standard.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONNECTIONS**
Kelly solved the following questions by connecting factoring, equation-solving skills, and operations on fractions to rational expressions and equations.

Numerical Response

1. The largest non-permissible value of x for the rational expression $\frac{x+4}{x^2+x-6}$ is ____2____.

(Record your answer on the answer sheet.)

1. The sum of $\frac{2}{x+1} + \frac{3}{x-2}$, where $x \neq -1$ or 2, is

A. $5x - 1$

B. $\frac{5}{(x+1)(x-2)}$

***C.** $\frac{5x-1}{(x+1)(x-2)}$

D. $\frac{5}{2x+1}$

The student demonstrating excellent achievement can answer the following type of question.

1. A simplified form of $\frac{c^2-4c+3}{2c-2} \div \frac{c^2-9}{4c^2-11c-3}$, where $c \neq -3, -\frac{1}{4}, 1$, or 3, is

A. $\frac{4c-1}{2}$

B. $4c^2 + 11c$

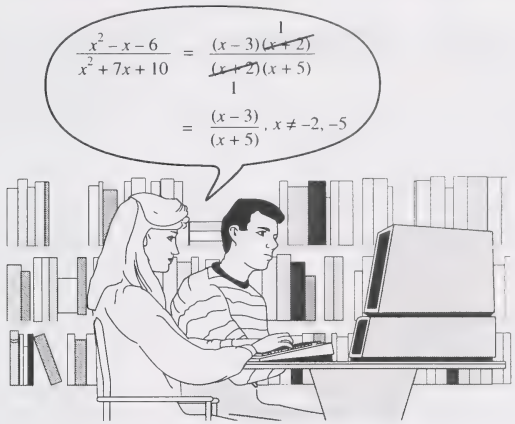
***C.** $\frac{(4c+1)(c-3)}{2(c+3)}$

D. $\frac{(c+3)(c+9)}{2(4c+3)}$

The student demonstrating acceptable and excellent achievement can answer part or all of the following question.

Use the following information to answer the next question.

Anita and Bobbie used the relationship between factoring and rational expressions to solve problems.



Written Response – 6 marks

1. **a.** Show the steps Anita could use to simplify the rational expression $\frac{x^2 + 5x + 6}{x^2 + 2x - 3} \times \frac{x^2 - x}{x^2 - 4}$.

POSSIBLE SOLUTION:

$$\frac{x^2 + 5x + 6}{x^2 + 2x - 3} \times \frac{x^2 - x}{x^2 - 4}$$

$$= \frac{(x+3)(x+2)}{(x+3)(x-1)} \times \frac{x(x-1)}{(x-2)(x+2)} = \frac{x}{x-2},$$

$x \neq -3,\ 1,\ -2$ and 2

b. Now, evaluate the simplified expression in part **a** for $x = 3$.

POSSIBLE SOLUTION:

When $x = 3$, $\frac{x}{x-2} = \frac{3}{3-2} = \frac{3}{1} = 3$

Use this additional information to answer the next question.

Next, Anita substituted into the original rational expression to determine a value.

Anita's substitution: $\frac{3^2 + 5(3) + 6}{3^2 + 2(3) - 3} \times \frac{(3)^2 - 3}{(3)^2 - 4}$

c. Explain whether or not Anita should expect the same answer as you obtained in part **b**. Provide supporting evidence for your explanation.

POSSIBLE SOLUTION:

Anita should expect the same answer:

Anita's Method: $= \frac{(3)^2 + 5(3) + 6}{(3)^2 + 2(3) - 3} \times \frac{(3)^2 - 3}{(3)^2 - 4}$

$= \frac{9 + 15 + 6}{9 + 6 - 3} \times \frac{9 - 3}{9 - 4}$

$= \frac{30}{12} \times \frac{6}{5} = \frac{180}{60} = 3$

My Method: $\frac{x}{x-2}$ is the simplified form for $\frac{x^2 + 5x + 6}{x^2 + 2x - 3} \times \frac{x^2 - x}{x^2 - 4}$, so $\frac{(3)}{(3) - 2} = 3$

$\therefore$ Both methods yield a result of 3.

$\therefore$ When evaluated at $x = 3$ the value obtained by both methods is equal, so Anita should expect the same answer.

Trigonometry

General Outcome

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Solve problems that involve triangles and their applications.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can also:**
• recognize and give examples of the principal and other co-terminal angles for angles drawn in standard position on a coordinate plane	• determine an expression for any angle that will yield all its possible co-terminal angles
• determine the measure of the reference angle for any angle in standard position on a coordinate plane	• determine the sine, cosine, and tangent ratios of an angle in standard position given the coordinates of any point, including points with literal coordinates, on its terminal arm
• determine the sine, cosine, and tangent ratios, and the measure of the principal angle for coordinates of any point with numerical coordinates on the terminal arm of an angle in standard position	• determine any two of the x-coordinate of a point on the terminal arm of an angle in standard position, the y-coordinate of a point on the terminal arm of an angle in standard position, the measure of the angle, and the distance that the point lies from the origin, given the other two
• determine the sine, cosine, and tangent ratios for any angle measure	• determine the measures of all angles between $0°$ and $360°$ that satisfy a given sine, cosine, or tangent ratio
• recognize the graph of $y = \sin \theta$	
• use the graph of $y = \sin \theta$ to solve trigonometric equations	
• describe, orally or in writing, the effects of changing either of the parameters a or b on the graph of the function $y = a \sin b\theta$	• describe, orally or in writing, the effects of changing both a and b on the graph of the function $y = a \sin b\theta$
• determine the measures of sides and angles in problems that involve two right triangles if a diagram is given	• determine the measures of sides and angles in problems that involve right triangles in two dimensions
	• given the diagram of three-dimensional objects composed of two or more right triangles, determine the missing measures of sides and angles of the triangles
• decide whether or not the information provided for a triangle uniquely defines an oblique triangle, and if it does not, construct the possible triangles	
• determine the missing measures of sides and angles in problems that involve oblique triangles if a diagram is given	
• determine the missing measures of sides and angles in problems that involve a uniquely defined oblique triangle if a diagram is given	• determine the missing measures of sides and angles in problems that involve oblique triangles

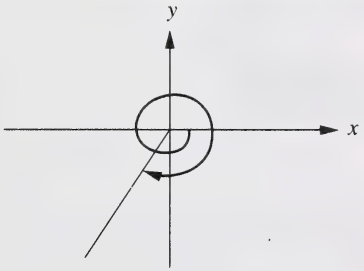
Illustration of the general outcome
Extend right angle trigonometry to angles in a coordinate plane.
Use understanding of sine and cosine law to solve oblique triangles.

The following are examples of questions from the Trigonometry Unit that illustrate the standard.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **TECHNOLOGY**
Yin and Henry used a graphing calculator to assist in analyzing, interpreting, and exploring equations and their graphical representations.

1. An angle in standard position is shown below.



The measure of the principal angle could be

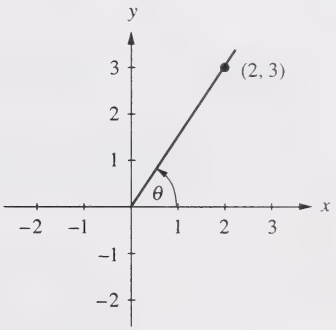
***A.** 220°
B. –140°
C. 140°
D. –580°

2. If $\tan\theta = -\frac{5}{12}$ and $0° < \theta < 180°$, then $\sin\theta$ equals

***A.** $\frac{5}{13}$
B. $\frac{13}{12}$
C. $\frac{12}{13}$
D. $\frac{5}{17}$

Use the following information to answer the next question.

Yin used a computer to plot the point (2, 3) on the terminal arm of angle θ.



3. The exact value of the ratio for $\cos\theta$ is

***A.** $\frac{2}{\sqrt{13}}$
B. $\frac{\sqrt{13}}{2}$
C. $\frac{3}{\sqrt{13}}$
D. $\frac{\sqrt{13}}{3}$

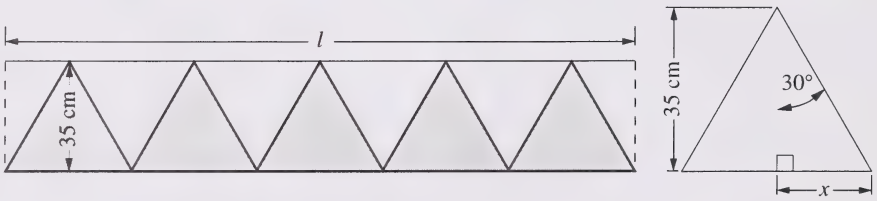
The student demonstrating excellent achievement can answer the following type of question.

SCENARIO: **ALBERTA INDUSTRIES**

Contractors, manufacturers, technologists, and construction workers constantly solve problems using Mathematics 33 skills.

Use the following information to answer the next question.

A manufacturing plan for a floor support is shown below. Each of the five shaded triangles is equilateral and has a height of 35 cm (an enlargement of one is shown on the right).



1. The length l of the floor support is

A. 101.0 cm
B. 175.0 cm
***C.** 202.1 cm
D. 210.1 cm

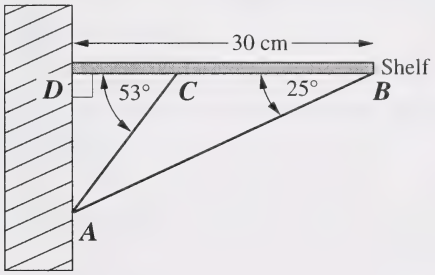
The student demonstrating acceptable and excellent achievement can answer part or all of the following question.

SCENARIO: **CROSS-CURRICULAR CONNECTIONS**

Some skills gained in Mathematics 33 are extensions of understanding from previous courses and can be applied to other courses.

Use the following information to answer the next question.

Lori built the triangular support system shown below.



Written Response — 5 marks

1. Lori built a shelf 30 cm in length. In order to keep it secure, Lori built two supports, $\overline{AB}$ and $\overline{AC}$, as shown in the diagram. Find the lengths, to the nearest tenth of a centimetre, of both supports, and show how you obtained your answers mathematically.

POSSIBLE SOLUTION:

Method I

The supports are $\overline{AB}$ and $\overline{AC}$. In order to find the measures of these sides, the side $\overline{DA}$ must be determined.

Using ΔADB, substitute $\angle DBA = 25°$

and $\overline{DB} = 30$ cm into tangent $\angle DBA = \dfrac{\overline{DA}}{\overline{DB}}$

$$\tan 25° = \frac{\overline{DA}}{30}$$

$$\overline{DA} = 30 (\tan 25°)$$

$$\therefore \overline{DA} = 13.9892 \text{ cm}$$

Now, use ΔCDA to find support $\overline{CA}$

$$\sin 53° = \frac{13.9892}{\overline{AC}}$$

$$\overline{AC} = 17.52 \text{ cm}$$

To determine $\angle ACB$:

$\angle ACB = 180° - 53° = 127°$

To determine the support $\overline{AB}$:

$$\frac{b}{\sin b} = \frac{c}{\sin c} \qquad \sin 25° = \frac{13.9892}{\overline{AB}}$$

or

$$\frac{17.52}{\sin 25°} = \frac{c}{\sin 127°} \qquad \overline{AB} = 33.1 \text{ cm}$$

$c = 33.1$ cm

The lengths of the supports $\overline{AC}$ and $\overline{AB}$ are 17.5 cm and 33.1 cm, respectively.

POSSIBLE SOLUTION:

Method II

Solving for $\overline{DA}$, use ΔADB and the tangent ratio for $\angle DBA$

tangent $\angle DBA = \dfrac{\overline{DA}}{\overline{DB}}$.

By substituting, we obtain $\tan 25° = \dfrac{\overline{DA}}{30}$

$\overline{DA} = 30(\tan 25°)$

$\therefore \overline{DA} = 13.9892$ cm

Using Pythagorean theorem to solve for $\overline{AB}$,

$(\overline{AB})^2 = (\overline{DA})^2 + (\overline{DB})^2$

$(\overline{AB})^2 = (13.9892)^2 + (30)^2$

$(\overline{AB})^2 = 1095.698549$

$\overline{AB} = 33.1$ cm

To solve for support $\overline{AC}$, use ΔADB and the sine ratio for $\angle DCA$.

$(\overline{AB})^2 = 195.6985488 + 900$

By substituting, we get

$\sin 53° = \dfrac{\overline{DA}}{\overline{AC}}$

$\sin 53° = \dfrac{13.9892}{\overline{AC}}$

$\overline{AC} = \dfrac{13.9892}{\sin 53°}$

$\therefore \overline{AC} = 17.5$ cm

The lengths of the supports $\overline{AC}$ and $\overline{AB}$ are 17.5 cm and 33.1 cm, respectively.

POSSIBLE SOLUTION:

Method III

To determine the measure of support $\overline{AB}$, use ΔABD and the cosine ratio for $\angle DBA$.

cosine $\angle DBA = \dfrac{\overline{DB}}{\overline{AB}}$

By substituting $\angle DBA = 25°$ and $\overline{DB} = 30$ cm, we obtain

$$\cos 25° = \frac{30}{\overline{AB}}$$

$$\overline{AB} = \frac{30}{\cos 25°}$$

$$\therefore \overline{AB} = 33.1 \text{ cm}$$

To determine the measure of support $\overline{AC}$, use ΔABC and the law of sines.

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

By substituting $\angle B = 25°$, $c = 33.1$ cm, and $\angle C = 180° - 53° = 127°$,

we obtain:

$$\frac{b}{\sin 25°} = \frac{33.1}{\sin 127°}$$

$$b = \frac{33.1 \times \sin 25°}{\sin 127°}$$

$$b = 17.52 \text{ cm}$$

$$\therefore \overline{AC} = b = 17.52 \text{ cm}$$

The lengths of the supports $\overline{AC}$ and $\overline{AB}$ are 17.5 cm and 33.1 cm, respectively.

Statistics

General Outcome

Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Collect, organize, analyze, and draw inferences from bivariate data and from data obtained through "yes/no" surveys.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can also:**
• collect and plot bivariate data	
• design and administer surveys, collect and organize results, and draw inferences from surveys including bivariate data and yes/no questions	
• assess the strengths, weaknesses, and biases of samples	
• recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatter plot	
• plot a line of best fit on a scatter plot using the median fit method	• develop and use prediction equations for a line of best fit to make inferences for populations
• draw 50% and 90% box plots of the results of multiple samples	
• use charts of 90% box plots to determine the confidence interval of the proportion of yeses in the population	• describe, orally or in writing, what is meant by the confidence interval for the proportion of yeses in the population
• illustrate the steps or strategies that lead to the solution of problems that require the analysis of statistics studied in Mathematics 33	• complete the solution to problems that require the analysis of statistics studied in Mathematics 33

Illustration of the general outcome

Given a problem whose solution requires the use of statistics, students should be able to design and administer surveys, collect and organize the results of surveys, draw inferences from surveys including bivariate data and yes/no questions, and determine the confidence intervals for the results of yes/no surveys.

The following are examples of questions from the Statistics Unit that illustrate the standard.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **SPORTS AND RECREATION**

Management decisions related to running a sports franchise are often linked to statistics. In the next question, the management would like you to draw an inference about the percentage of patrons who will buy hot dogs.

Numerical Response

1. Out of a sample of 20 randomly selected patrons at a baseball stadium, 8 bought a hot dog during the game. Based on this sample and using a 90% box plot, the percentage of all patrons at the baseball stadium buying a hot dog is between 25% and ____60____ %.

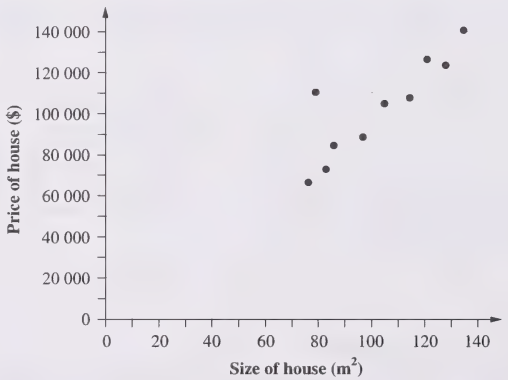
(Record your answer on the answer sheet.)

SCENARIO: **ALBERTA INDUSTRIES**

Contractors, manufacturers, technologists, and construction workers constantly solve problems using Mathematics 33 skills.

Use the following information to answer the next question.

The contractor used the scatter plot shown below to determine if there was a correlation between the price and the size of the house built.



According to this scatter plot, as the size of house __*i*__, the price of house __*ii*__.

1. Which row on the chart correctly completes the statement related to the scatter plot above?

Row	*i*	*ii*
*A.	decreases	decreases
B.	increases	decreases
C.	increases	remains constant
D.	decreases	remains constant

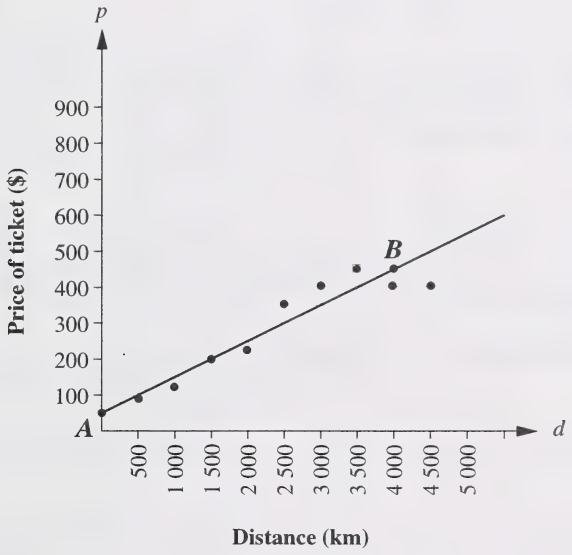
The student demonstrating excellent achievement can answer the following type of question.

SCENARIO: **AIRLINE INDUSTRY**

Workers in the airline industry use their mathematical understanding to solve work-related problems and problems associated with everyday life.

Use the following information to answer the next question.

The airline company used the scatter plot shown below to illustrate the relationship between the price, p, in dollars of a one-way ticket and the distance, d, in kilometres of the flight.



1. If two of the points on the line of best fit are $A(0, 50)$ and $B(4\,000, 450)$, then the equation of the line of best fit is

***A.** $p = \frac{1}{10}d + 50$

B. $p = 10d + 50$

C. $p = \frac{1}{10}d$

D. $p = 50$

The student demonstrating acceptable and excellent achievement can answer part or all of the following question.

SCENARIO: **SPORTS AND RECREATION**

As a class project, mathematics 33 students were asked to make connections between the mathematics curriculum studied and activities related to physical education.

Use the following information to answer the next question.

Pauline's project included a statistical comparison of the Mathematics 33 and Physical Education 30 exam marks for 10 selected students.

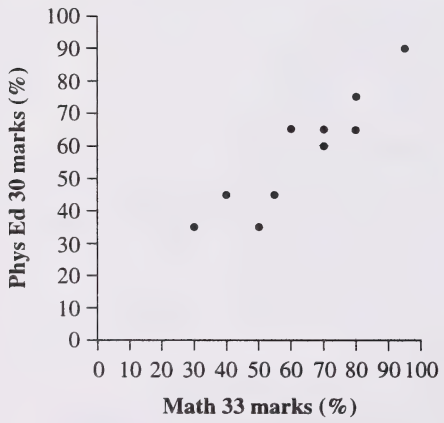
The table summarizes Pauline's findings.

Student	1	2	3	4	5	6	7	8	9	10
Math 33 Marks (%)	30	70	95	80	60	40	50	55	70	80
Phys. Ed. Marks (%)	35	60	90	75	65	45	35	45	65	65

Written Response — 6 marks

1. **a.** On the grid provided below, draw a scatter plot for the above data.

POSSIBLE SOLUTION:



(These points must be plotted accurately.)

b. i. Describe the apparent correlation between a student's Mathematics 33 mark and his or her Physical Education 30 mark, stating the strength and direction of the correlation.

ii. From this correlation, make a correct inference that begins, "As Mathematics 33 marks increase, ..."

POSSIBLE SOLUTION:

i. The correlation between a student's Mathematics 33 and Physical Education 30 marks is positive and strong.

ii. As Mathematics 33 marks increase, Physical Education 30 marks also increase.

c. In theory, what approximate mark would a student get in Physical Education 30 if he or she got 20% in Mathematics 33? **Explain** how you arrived at your value.

POSSIBLE SOLUTION:

By plotting a line of best fit and extrapolating, a Physical Education mark of 15% can be obtained for a student who gets a Mathematics 33 grade of 20%.

Annuities, Mortgages, and Loans

General Outcome
Students who achieve the standards can demonstrate the following general outcome upon completion of the unit: *Apply the concepts of ratio, rate, percentage, and proportion to annuities, mortgages, and loans.*

The student demonstrating acceptable achievement can:	**The student demonstrating excellent achievement can:**
• recognize that an annuity is a sequence of payments paid in regular intervals for purposes of investment or income	• given a situation, identify the situation as a present value of an annuity or an amount of an annuity, and then solve a related problem
• recognize that a loan is a sum of money that is borrowed to purchase items and is usually repaid in a sequence of payments	• given a situation, identify it as a situation where a loan, annuity, or mortgage would be necessary, and solve the related problem
• recognize that a mortgage is a special type of loan used to purchase real estate	
• given the principal, interest rate, and term, determine the monthly payment for a loan or mortgage	• given the principal, interest rate, and term, find the monthly payment of a loan or mortgage and the interest or the total amount to be repaid
• solve problems that are clearly identified as mortgage, loan, or annuity problems	
• given the periodic payment, interest rate, and term, determine the amount of an annuity	• given the total amount of an annuity, the interest rate, and term, determine the periodic payment
• given the periodic payment, number of payments, and interest rate, determine the present value of an annuity	• given the present value of an annuity, number of payments, and interest rate, determine the periodic payments
• show that the amount or present value of an annuity is determined by multiplying the amount of a regular payment by a value in a table	
• show how the amount or present value of an annuity can be determined, or how a loan or mortgage can be paid off, using a calculator or spreadsheet	

Illustration of the general outcome
Extend understanding of rate, ratios, percentage, and proportion to solve real-world problems related to annuities, mortgages, and loans.

The following are examples from the Annuities, Mortgages, and Loans Unit that illustrate the standard.

The student demonstrating acceptable achievement can answer the following types of questions.

SCENARIO: **CONSUMERISM**

Dawn is a financial advisor. She constantly uses mathematics to make decisions related to her customers' investment and banking opportunities. Answer the following questions related to Dawn's experiences.

Use the following information to answer the next question.

In order to save money in an annuity, Dawn advised Mr. and Mrs. Bartel to invest $500.00 at the beginning of every month for 28 months.

1. If the investment earns $1\frac{1}{2}\%$ per month, the amount of the annuity at the end of the 28 months will be

A. $11 363.36
B. $14 000.00
C. $16 740.74
***D.** $17 499.35

Use the following information to answer the next question.

Dawn arranged a $120 000 mortgage for the Kim family. The Kims mortgaged their new home for 10 years at a rate of $9\frac{1}{2}\%$ per annum.

Numerical Response

1. The Kims' monthly payment, to the nearest dollar, will be $\$\_\_1\,541\_\_$.
(Record your answer on the answer sheet.)

Use the following information to answer the next two questions.

Dawn organized an annuity for another customer. Starting in January, every six months, the customer will deposit $1 000 into an annuity that pays interest at 6% per annum, compounded semi-annually. Entries related to this annuity, for a 2-year period, are shown below.

Payment Period	Regular Payment	New Balance	Interest per Period	Final Balance
1	$1 000	$1 000.00	$1 000 × 0.03 = $30.00	$1 030.00
2	$1 000	$2 030.00	$2 030 × 0.03 = $60.90	$2 090.90
3	$1 000	$3 090.90	*i*	.....
4	$1 000	.....	*ii*	*iii*

2. The value of ***i***, the interest for payment period 3, is

***A.** $92.73
B. $91.80
C. $90.90
D. $90.30

Numerical Response

2. To the nearest dollar, the value of the final balance ***iii*** is $_4 309_.
(Record your answer on the answer sheet.)

The student demonstrating excellent achievement can answer the following type of question.

Use the following information to answer the next question.

A customer borrowed $14 700 at 15.75% per annum, compounded monthly, and made monthly payments of $515.

Numerical Response

1. The number of months it will take to repay the $14 700 is _36_ months.
(Record your answer on the answer sheet.)

The student demonstrating acceptable and excellent achievement can answer part or all of the following question.

SCENARIO: **AIRLINE INDUSTRY**

Workers in the airline industry use their mathematical understanding to solve work-related problems and problems associated with everyday life.

Use the following information to answer the next question.

Sydney borrowed \$2 000.00 at 12% per annum compounded monthly to buy a car. Sydney's payments are \$178.00 monthly for one year, with the interest calculated monthly. At the end of one year, the loan is fully paid off. To analyze the loan payments, Sydney set up the following amortization spreadsheet and made the first several entries.

Monthly Payment Periods	Amount Paid/Month	Interest Paid/Period	Principal Paid	Outstanding Balance
0				\$2 000.00
1	\$178.00	\$2 000.00 × 0.01 = \$20.00	\$178.00 – \$20.00 = \$158.00	\$1 842.00
2	\$178.00	\$1 842.00 × 0.01 = \$18.42	\$178.00 – \$18.42 = \$159.58	\$1 682.42
3	\$178.00	*A*	\$161.18	\$1 521.24
4	\$178.00	*B*	*C*	*D*

Written Response — 5 marks

1. **a.** Determine *A*, *B*, *C*, and *D* in the amortization table, and show how you obtained these values.

POSSIBLE SOLUTION:

The value for cell *A* (**Interest Paid/Period**) is determined by:
Amount Paid/Month – Principal Paid = Interest Paid/Period
\$178.00 – \$161.18 = \$16.82
or
Outstanding Balance from Previous Period × Rate of Interest/Period = Interest Paid/Period
\$1682.42 × 0.01 = \$16.82

The value for cell *B* (**Interest Paid/Period**) is determined by:
Outstanding Balance from Previous Period × Rate of Interest/Period = Interest Paid/Period
\$1 521.24 × 0.01 = \$15.21

The value for cell *C* (**Principal Paid**) is determined by:
Amount Paid/Month – Interest Paid/Period = Principal Paid
\$178.00 – \$15.21 = \$162.79

The value for cell *D* (**Outstanding Balance**) is determined by:
Previous Outstanding Balance – Principal Paid = New Outstanding Balance
\$1 521.24 – 162.79 = \$1 358.45

b. Find the total amount of interest paid in the first year, and show how you determined it.

POSSIBLE SOLUTION:

Method I

The total amount of interest paid for one year can be estimated by calculating the total amount paid in one year and subtracting \$2 000.00.

$$\therefore \$178.00 \times 12 = \$2\ 136.00$$
$$\$2\ 136.00 - \$2\ 000.00 = \$136.00$$

Sydney paid \$136.00 interest for the first year.

Method II

The total amount of interest paid for one year can be estimated by calculating the interest paid at each period and obtaining the total of all interest payments.

Period	Payment	Interest	Principal	Balance
0				2 000.00
1	178.00	20.00	158.00	1 842.00
2	178.00	18.42	159.58	1 682.42
3	178.00	16.82	161.18	1 521.24
4	178.00	15.21	162.79	1 358.45
5	178.00	13.58	164.42	1 194.03
6	178.00	11.94	166.06	1 027.07
7	178.00	10.28	167.72	860.25
8	178.00	8.60	169.40	690.85
9	178.00	6.91	171.09	519.76
10	178.00	5.20	172.80	346.96
11	178.00	3.47	174.53	172.43
12	178.00	1.72	176.28	–3.85

The total interest is \$132.15 ± \$1.00.

Method III

Students could use the data booklet.

$$\$88.8488 \times 2 \times 12 = \$2\ 132.37$$
$$\$2\ 132.17 - \$2\ 000.00 = \$132.37$$

Note: Accept \$132.37 or \$132.38 or \$132.40, due to the different ways of rounding off \$88.8488.

Appendix C

Explanation of Mathematical Understandings

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate
- give reasons for the steps in a procedure
- reliably and efficiently execute procedures
- verify the results of procedures empirically (e.g., using models) or analytically
- recognize correct and incorrect procedures
- generate new procedures and extend or modify familiar ones
- appreciate the nature and role of procedures in mathematics

It is important that students know how to execute mathematical procedures reliably and efficiently; a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics; that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures: it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts
- identify and generate examples and non-examples
- use models, diagrams, and symbols to represent concepts
- translate from one mode of representation to another
- recognize the various meanings and interpretations of concepts
- identify properties of a given concept and recognize conditions that determine a particular concept
- compare and contrast concepts

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples; it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems
- apply a variety of strategies to solve problems
- solve problems
- verify and interpret results
- generalize solutions

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

Appendix D

Directing Words

The following words are specific in meaning.

Compare
Show the character or relative values of two things by pointing out their *similarities* and *differences*

Conclude
State a logical end based on reasoning and/or evidence

Contrast/Distinguish
Point out the *differences* between two things that have similar or comparable natures

Criticize
Point out the *merits* and *demerits* of an item or issue

Define
Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits

Describe
Give a written account or represent the characteristics of something by a figure, model, or picture

Design/Plan
Construct a plan, i.e, a detailed sequence of actions, for a specific purpose

Enumerate
Specify one by one or list in concise form and according to some order

Evaluate
Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages

Explain
Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail

How
Show in what manner or way, with what meaning

Hypothesize
Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically

Identify
Recognize and select as having the characteristics of something

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence

Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data

Justify/Show How
Show reasons for or give facts that support a position

Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps

Predict
Tell in advance on the basis of empirical evidence and/or logic

Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons

Relate
Show logical or causal connection between things

Solve
Give a solution for a problem; i.e., explanation in words and/or numbers

Summarize
Give a brief account of the main points

Trace
Give a step-by-step description of the development

Why
Show the cause, reason, or purpose

Appendix E

Answer Sheet

Following is a copy of the answer sheet that will be used for the 1997-98 school year Mathematics 33 diploma examinations. Teachers are encouraged to copy and use the answer sheet for their own examinations in order to familiarize students with it.

MULTIPLE CHOICE

1 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ	33 Ⓐ Ⓑ Ⓒ Ⓓ	41 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ	34 Ⓐ Ⓑ Ⓒ Ⓓ	42 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ	35 Ⓐ Ⓑ Ⓒ Ⓓ	43 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ	28 Ⓐ Ⓑ Ⓒ Ⓓ	36 Ⓐ Ⓑ Ⓒ Ⓓ	44 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ	29 Ⓐ Ⓑ Ⓒ Ⓓ	37 Ⓐ Ⓑ Ⓒ Ⓓ	45 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ	30 Ⓐ Ⓑ Ⓒ Ⓓ	38 Ⓐ Ⓑ Ⓒ Ⓓ	46 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ	31 Ⓐ Ⓑ Ⓒ Ⓓ	39 Ⓐ Ⓑ Ⓒ Ⓓ	47 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ	32 Ⓐ Ⓑ Ⓒ Ⓓ	40 Ⓐ Ⓑ Ⓒ Ⓓ	48 Ⓐ Ⓑ Ⓒ Ⓓ

NUMERICAL RESPONSE

1	2	3	4	5	6	7
⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

8	9	10	11	12	13	14
⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙	⊙ ⊙
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Appendix F

Instructions for Completing the Multiple-Choice, Numerical-Response, and Written-Response Questions

Multiple Choice

- Decide which of the choices **best** completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

Example

This examination is for the subject of

A. mathematics
B. chemistry
C. biology
D. physics

Answer Sheet

● Ⓑ Ⓒ Ⓓ

Numerical Response

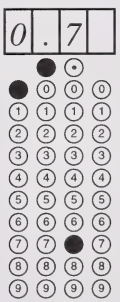
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- If an answer is a value between 0 and 1 (e.g., 0.25), then be sure to record the 0 before the decimal place.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**

Example 1

The value of tan 35° to the nearest tenth is _____.
(Record your answer on the answer sheet.)

Calculator value: 0.7002075
Value to be recorded: 0.7

Record 0.7 on the answer sheet →



Example 2

The constant term in the quadratic function $y = 2x^2 + 7x + 32$ is _____.
(Record your answer on the answer sheet.)

Value to be recorded: 32

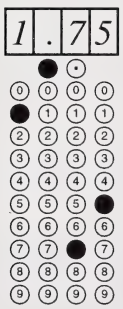
Record 32 on the answer sheet →

Example 3

If an annual interest rate of 7% is compounded quarterly, then the quarterly rate to the nearest hundredth of a percent is _____%.
(Record your answer on the answer sheet.)

Value to be recorded: 1.75

Record 1.75 on the answer sheet →



Written Response

- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers must be well organized and address **all** the main points of the question.
- Descriptions and/or explanations of concepts must be correct and reflect pertinent ideas, calculations, and formulas.
- Your answers **should be** presented in a well-organized manner using complete sentences, and correct units.

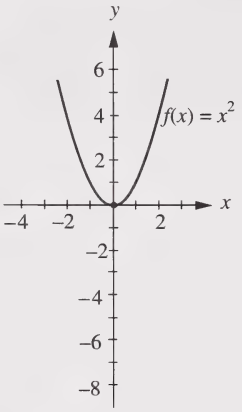
Appendix G

Sample Written-Response Question and Scoring Guide

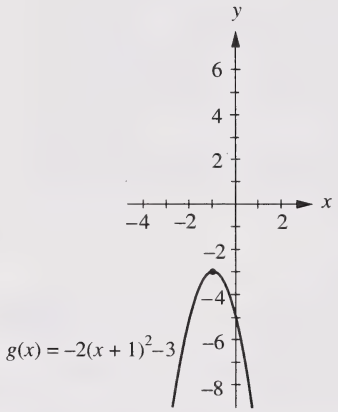
Use the following information to answer the next question.

A graphics designer transformed the graph of the quadratic function $f(x) = x^2$, shown in graph I below, into the graph of the quadratic function $g(x) = -2(x + 1)^2 - 3$, shown in graph II below.

Graph I



Graph II



Written Response – 5 marks

1. **a.** **Refer to the functions** provided to explain **three** major changes that occurred when graph I, which represents $f(x) = x^2$, was transformed into graph II, which represents $g(x) = -2(x + 1)^2 - 3$.

A POSSIBLE SOLUTION - Part a

Using the general form $y = a(x - h)^2 + k$ to compare graphs I and II, the parameter

- 'a' in graph I is positive and therefore the graph opens upward whereas in graph II, $a = -2$ which causes the graph to open downward. The absolute value of $a_2 > a_1$, therefore graph II is vertically expanded.
- 'h' in graph I is 0 whereas in graph II it is equal to positive one transforming graph II one unit to the left (the axis of symmetry shifts left). The vertex of graph I is at (0, 0), whereas in graph II, the vertex has shifted to (–1, –3) **or** The equation of the axis of symmetry ($x = h$) has changed from $x = 0$ to $x = -1$.
- 'k' has a value of zero and graph I passes through the origin, whereas in graph II, $k = -3$, transforming graph II down 3 units.

b. The range for the graph of the function $f(x) = x^2$ is $y \geq 0$. The range for the graph of the function $g(x) = -2(x + 1)^2 - 3$ is $y \leq -3$. Write a quadratic function for a graph with a vertex located in quadrant I that has a range of $y \geq 2$. Begin with $h(x) =$.

- $h(x) =$
- Provide a written explanation why the function you wrote has a range of $y \geq 2$. Your explanation must include more than a sketched graph.

A POSSIBLE SOLUTION

The equation can vary and be of the form $h(x) = a(x - h)^2 + k$, where $a > 0$, $h > 0$ and $k = 2$. To have a range of $y \geq 2$, "a" must be positive to get an upwards oriented graph, h must be a positive number to shift the vertex to the right, into quadrant I and k must equal 2 so the graph reaches a minimum at $(h, 2)$.

The graph of this function would have a vertex of $(h, 2)$ and be directed upwards. Since 2 is a minimum y-value for the graph, the range would be $y \geq 2$.

The following represents a generalized scoring criteria that was adapted to the previous question. The scoring criteria is very similar to that used for all 5 mark questions.

This scoring criteria is based on a five-point scale.

Marks	Descriptor
5	The student • correctly completes **the** solution with supporting detail **and** • provides and **clearly** communicates reasons in logical order
4	The student • provides a **complete and correct** answer except for a minor error present
3	The student • provides a partial answer that is complete as far as it goes and represents a **major** step in the solution to the problem **or** • provides a **complete** answer that has one major or many minor errors but does indicate the intended scope of the problem **and** • provides reasonable communication
2	The student • provides a **partial** answer that is complete and correct **and** • provides limited communication
1	The student • makes a significant start (holistically) which could have lead to a correct solution to the problem **or** • provides a correct answer with no supporting work **or** • provides a correct explanation
0	The student • makes no significant start to show any relevant understanding • makes an attempt to solve the problem with no relevant work evident

Minor Error:
• logic/syntax error
• communication error

Major Error:
• incorrect understanding of the transformational effects of the parameters.
• incorrect understanding of range

Major Step:
• a complete answer to a or b
(a clear understanding of the parameters a, h, k)
• explanations are quantified with reference to the equation